Librational Instability of Rigid Space Station Due to Translation of Internal Mass

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The attitude stability of an Earth-pointing space station subjected to harmonic translation of onboard equipment is considered. It is shown that instability may occur if the period of translation is in the vicinity of the orbital period. Such periodic motion may result, for example, from experimentation that would require mass or crew shifting to the "Earth end" of the station at each passover of a ground location. Instability occurs for a range of station configurations expressed parametrically as two principal-moment-of-inertia ratios. The larger the moving internal mass and the larger its amplitude of motion, the wider the region of instability. Results are presented for point mass motion along the station axis that nominally is aligned to the local vertical. A similar analysis procedure could be applied to translation of a mass of arbitrary shape along any principal axis.

Nomenclature

a_T	= constant coefficient of Hill's equation
$\mathbf{b}_1,\mathbf{b}_2,\mathbf{b}_3$	= unit vectors along body principal axes
$\mathbf{h}$	= angular momentum vector of space station
$\mathbf{I}$	= inertia dyadic of the space station
I_1,I_2,I_3	= principal moments of inertia of the space station about composite center of mass
I_{f1},I_{f2},I_{f3}	= principal moments of inertia of the space station excluding the moving mass
L	= double amplitude of point mass harmonic translation
$\mathbf{M}$	= gravitational torque about station center of mass
M	= reduced mass of moving element
m_m	= mass of moving element
m_f	= mass of fixed portion of space station
n	= orbital rate for circular orbits
$\mathbf{o}_1,\mathbf{o}_2,\mathbf{o}_3$	= unit vectors along axes of orbital frame aligned to binormal, tangent and normal to orbital path
p,r,y	= pitch, roll, and yaw Euler angles
q_1,q_2,q_3,q_4	= constant coefficients of Hill's equation
$\mathbf{R}$ (R_1,R_2,R_3)	= radius from Earth center to space station center of mass
r_0	= ratio of moving element inertia to pitch inertia
r_1,r_2	= ratios of station principal moments of inertia
T	= dimensionless time variable
T_0,T_1	= time scales of method of multipliers
t	= time
u	= state vector of roll/yaw Euler angles
${}^I\omega^B$	= angular velocity of space station with respect to an inertial frame
z	= transformed pitch variable
$\alpha_1,\alpha_2,$	= time varying coefficients of roll/yaw equations
ϵ	= perturbation parameter
μ	= characteristic exponent in Hill's equation solution
ω	= frequency of motion of moving mass
ω_1,ω_2	= resonant frequencies of roll/yaw motion with no relative motion of the internal mass

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Introduction

THE attitude stability of a single rigid body in an inverse square gravitational field was first described by Lagrange in 1780 (Ref. 1 traces the early developments), and, with the advent of the space age, by Debra and Delp.[2] Stable equilibrium solutions were found with the body principal axes parallel to the binormal, tangent, and normal of the orbit path, and principal moment of inertia ratios were identified for two stable regions in configuration space. These regions are now commonly referred to as the Lagrange and Debra-Delp regions.[3] The former is Lyapunov stable, whereas the latter is only gyrically stable. Spacecraft flexibility causes the Delp region to lose its stable characteristics, so the practical design is limited to configurations defined by the Lagrange region (Ref. 3, p. 318).

Apparent instabilities within the Lagrange region were noted by Kane.[4] Breakwell and Pringle[5] identified nonlinear coupling of the rotational equations as the explanation, and noted that the apparent instabilities were actually bounded motions in which librational kinetic energy is transferred between axes in a controlled manner when certain frequency ratios exist in the linearized equations of motion. These "internal resonant conditions" were identified as subregions within the Lagrange and Debra-Delp regions that, for most applications, should be avoided.

The effect of orbital eccentricity on stability of an Earth-pointing spacecraft has also been examined.[5-7] Subregions within the Lagrange and Debra-Delp regions were identified by Breakwell and Pringle[5] where eccentricity produces unbounded motion through parametric excitation at certain "external resonant conditions."

Flight experience of the Russian Salyut-6 space station included gravity-gradient stabilization for a period of time to conserve thruster propellant (Ref. 3, p. 334). During this time, crew disturbance to attitude was noted. Skylab experience also noted the disturbance to attitude when crew members jogged.[8] The effect of internal mass translation has been investigated for spinning space stations[9,10] and for actively controlled attitude.[11] A generic formulation of attitude equations to include mass translation was recently developed.[12]

In this paper, the effect of internal mass translation on the stability of a gravity gradient stabilized station is examined. The following assumptions are made:

1) A station of arbitrary but rigid shape follows a circular orbit about a point source of inverse square gravitational attraction; there are no other external forces.

2) A concentrated mass is assumed to translate harmonically along the yaw axis of the station.

3) Linearized equations of librational motion are studied to predict instability, and nonlinear equations are numerically integrated to verify the prediction.

While pure harmonic motion of the internal mass is not likely to occur in practice, station activity that approximates such is conceivable; for example, the shifting of equipment or crew with orbital periodicity to observe a ground site or station keeping satellite. Harmonic translation is introduced to facilitate characterization of the response to a "known" parametric energy source, analogous to the more common linear systems procedure of frequency response characterization based on a harmonic forcing function. The results provide useful insight and trends regarding attitude disturbance to a passively stabilized station resulting from internal mass translation. Instability is predicted at resonant conditions distinct from, but similar to, the internal and external resonant conditions of nonlinear coupling and eccentricity, respectively.

Equations of Motion

Let unit vectors b_1, b_2, and b_3 be along the axes of a frame that remain parallel to the space station principal axes. In their nominal and unperturbed orientation, they are parallel to the correspondingly labeled unit vectors of the orbital or o-frame (Fig. 1). It is convenient to place the origin of the b-frame at the composite center of mass, even though its location within the station then varies as the internal mass translates.

Considering only relative, harmonic translation along the longest or b_1 axis, the station moments of inertia about the b-frame axes can be expressed as

$$I_1 = I_{f1} \tag{1a}$$

$$I_2 = I_{f2} + M[(L/2)\ \sin(\omega t)]^2 \tag{1b}$$

$$I_3 = I_{f3} + M[(L/2\ \sin(\omega t)]^2 \tag{1c}$$

where I_{fi} is the ith principal moment of inertia about the center of mass of the fixed part of the station, disregarding the moving mass, ω is the circular frequency of oscillation of the moving mass, and L its double amplitude of travel. M is the "reduced mass" defined by

$$M = \frac{m_m m_f}{m_m + m_f} \tag{2}$$

where m_m is the magnitude of the moving mass and m_f is the magnitude of the fixed station mass. Note that relative motion along either of the other two principal axes would cause the inertia on its axis to be time invariant and would require relocation of one of the time varying terms in Eq. (1). Simultaneous motion on two axes, not considered in this paper, would change principal axes orientation in the station and require introduction of product of inertia terms.

Two assumptions are made that cause the composite center of mass to maintain a circular orbit: 1) the only external force present is gravity, model led as an inverse-square force field from a point source, and 2) station dimensions are small relative to orbit radius so that higher order terms in the gravity-gradient force representation that are capable of coupling orbital motion to attitude motion are ignored (Ref. 3, p. 292). Hence, both the fixed and moving portions of the station will "bob" about the circular path traced by the composite center of mass as the prescribed relative motion is applied. The identification of fixed and moving parts is only relevant in the sense that the fixed portion is typically larger and, therefore, exhibits less bobbing. Reduced mass ratios from zero to one-third are considered in this paper. Translational frequencies as low as one-half orbital frequency are considered.

The angular velocity of the station relative to an inertial frame is ${}^I\boldsymbol{w}^B$ and the angular momentum about the composite center of mass $\boldsymbol{h}$ can be written

$$\boldsymbol{h} = \boldsymbol{I} \bullet {}^I\boldsymbol{w}^B \tag{3}$$

where $\boldsymbol{I}$ is the inertia dyadic of the space station about the composite center of mass. The external moment $\boldsymbol{M}$ can be equated to the time rate-of-change of the station angular momentum

$$\boldsymbol{M} = \boldsymbol{I} \bullet \frac{{}^b\mathrm{d}}{\mathrm{d}t}[{}^I\boldsymbol{w}^B] + \frac{{}^b\mathrm{d}}{\mathrm{d}t}[\boldsymbol{I}] \bullet {}^I\boldsymbol{w}^B + {}^I\boldsymbol{w}^B \times [\boldsymbol{I} \bullet {}^I\boldsymbol{w}^B] \tag{4}$$

where the frame relative to which the derivative is taken is the b-frame, indicated by the presuperscript on the derivative.

The left-hand side of Eq. (4) can be expressed, under the above assumptions, as (see Ref. 3, p. 236)

$$\boldsymbol{M} = (3\mu/R^5)[\boldsymbol{R}x(\boldsymbol{I} \bullet \boldsymbol{R})] \tag{5}$$

where $\boldsymbol{R}$ is the radius vector from Earth center to the mass center of the spacecraft, R is the magnitude of $\boldsymbol{R}$, and μ is the Earth's gravitational constant. In b-frame components, Eq.

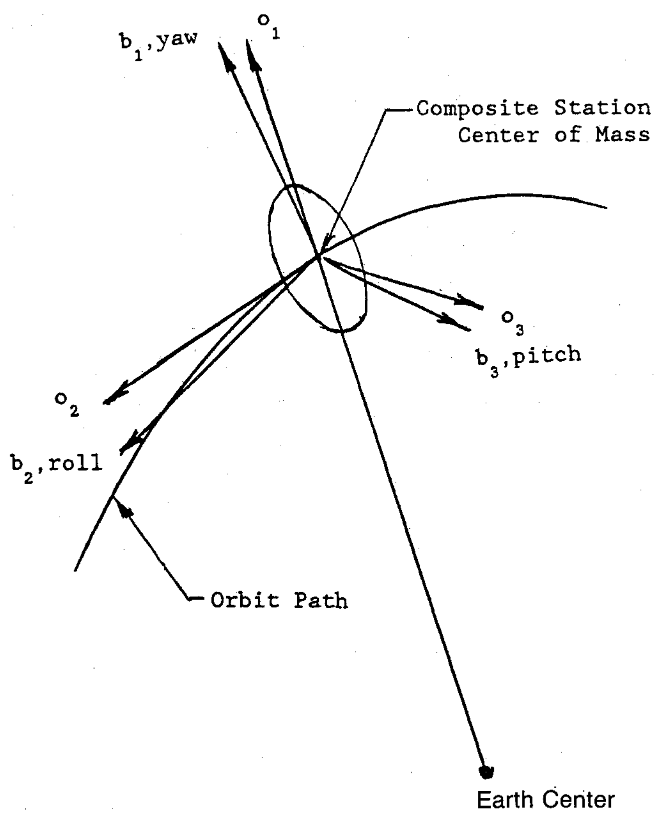

Fig. 1 Orbit and body axes.

(4) then becomes three coupled, nonlinear, ordinary differential equations

$$(3\mu/R^5)(I_3-I_2)R_2R_3=I_1\dot{w}_1+(I_3-I_2)w_2w_3 \tag{6a}$$

$$(3\mu/R^5)(I_1-I_3)R_1R_3=I_2\dot{w}_2+\dot{I}_2w_2+(I_1-I_3)w_3w_1 \tag{6b}$$

$$(3\mu/R^5)(I_2-I_1)R_1R_2=I_3\dot{w}_3+\dot{I}_3w_3+(I_2-I_1)w_1w_2 \tag{6c}$$

where the dot over an I or w term implies time rate of change of that variable.

Two features distinguish Eqs. (6) from those for libration of a single rigid body: 1) inertia parameters I_2 and I_3 vary with time and 2) additional terms $\dot{I}_2w_2$ and $\dot{I}_3w_3$ appear as a result of the time varying inertias. The effect of the time varying inertias is studied by linearizing Eqs. (6), using small angle approximations. A set of Euler rotations, p, y, and r about body axes b_3, b_1 and b_2, sequentially, that describe b-frame orientation relative to the o-frame, become the dependent variables in three linear equations with time varying coefficients:

$$I_1\ddot{y}+(I_3-I_2-I_1)n\dot{r}+(I_3-I_2)n^2y=0 \tag{7a}$$

$$I_2\ddot{r}+(I_2+I_1-I_3)n\dot{y}-4(I_1-I_3)n^2r+\dot{I}_2(\dot{r}+yn)=0 \tag{7b}$$

$$I_3\ddot{p}+3n^2(I_2-I_1)p+\dot{I}_3(\dot{p}+n)=0 \tag{7c}$$

where n is the orbital rate.

This paper studies the attitude motion of the station via Eqs. (7), when the internal mass moves as prescribed by Eq. (1). In particular, the objective is to identify station characteristics that might produce undesirable large amplitude motion. In contrast to internal resonance phenomena that are manifested only when first-order nonlinear terms are retained, internal mass motion is present as periodic coefficients in the linearized equations of motion and, hence, has the potential to be a more troublesome instability. The combined stability effects of internal or external resonance and mass translation are likely to be different than either considered separately and may be significant at high-mass ratios and certain inertia ratios. In this paper, only mass motion effects are considered. Note the linearized pitch equation (7c) remains uncoupled from roll and yaw as for the case of no moving mass developed by Hughes (Ref. 3, p. 295).

Stability

Equations (7) are valid when p, r, and y are sufficiently small that trigonometric functions containing them can be represented by the first term of their series expansion. The linear equations can be used to examine the trend of motion for small attitude perturbations. If, following a small attitude disturbance, the solution tends to zero or is bounded as $t\Rightarrow\infty$, the solution is said to be stable; if it tends toward $\mp\infty$ as $t\Rightarrow\infty$, it is said to be unstable.

This instability must, in general, be interpreted as an initial tendency. As any of the Euler angles grows beyond the small angle approximation, Eqs. (7) must give way to nonlinear Eqs. (6), where the stability conclusions may be quite different. However, the space station, with moving internal mass, is a nonconservative system. Some power source is required to produce the relative motion of the moving mass. Hence, the nonlinear equations would also be expected to exhibit instability since they include no dissipation. Numerical integration of the nonlinear equations is performed for some specific initial conditions to verify that the instability predicted by linearized analysis exists in the nonlinear equations.

Pitch Stability

By introducing the variable transformation

$$p=z\,e^{-\int P(t)dt} \tag{8}$$

the homogeneous equation resulting from Eq. (7c) is

$$z''+\left[a_T+16q_1\cos(2T)+16q_2\cos(4T)+16q_3\cos(6T)+16q_4\cos(8T)\right]z=0 \tag{9}$$

in which the independent variable has been changed from t to T through the definition $T=\omega t$, and the coefficients a_T, q_1, q_2, q_3, and q_4 are a function of station configuration parameters $r_0=ML^2/4I_{f3}$, $r_1=I_{f1}/I_{f3}$, and $r_2=I_{f2}/I_{f3}$ and frequency parameters n and ω. A series expansion of these coefficients that contains terms through order three in r_0 are shown in Table 1. Details of the development of Eq. (9), which is a form of Hill's equation, are given in Ref. 13.

The solution to Hill's equation, which is well studied,[14] is of the form

$$z=c_1e^{\mu T}\phi(T,\sigma)+c_2e^{-\mu T}\phi(T,-\sigma) \tag{10}$$

where $\phi(T,\sigma)$ and $\phi(T,-\sigma)$ are periodic functions and the stability of the system is determined by the characteristic exponent μ, which is a function of the a_T and the four q_i parameters (see Ref. 14, pp. 341, 342). The parameter five-space can be divided into regions of bounded and unbounded motions, with the periodic motions corresponding to $\mu=0$, providing surfaces that separate these regions. If we use configuration parameters, the unstable region can be portrayed graphically in terms r_1 and r_2 when r_0, ω, and n are specified. Thus, inertia ratios that correspond to the instability of internal mass translation can be superimposed on the Lagrange region, which is the locus of stable configurations when there is no internal mass translation. In Fig. 2, the locus of results for $\omega=n$ and $r_0=0.33$ is a pair of nearly straight lines labeled "Hill" that bound an unstable subregion within the Lagrange region.

The unstable region labeled "nonlinear" on Fig. 2 is produced by numerically integrating nonlinear Eqs. (6) with a 5 s integration time step and identifying as unstable any pitching motion which exceeded 90 deg within 1000 orbits. The unstable region labeled "Mathieu" is the region predicted by using a more restrictive assumption on the form of $\phi(T,\sigma)$ in Eq. (10) (see Ref. 14, p. 94), that, in essence, is comparable to only considering the effect of the a_T and q_1 terms in Eq. (9). These Mathieu boundaries are adequately represented by $a_t=1\pm q_1$, since the magnitude of q_1 is an order of magnitude

Table 1 Hill's equation coefficients

$$a_T=(9/8)(n/\omega)^2r_0^2(r_2-r_1-1)-(3/2)(n/\omega)^2r_0(r_2-r_1-1)+3(n/\omega)^2(r_2-r_1)+(15/16)(n/\omega)^2r_0^3+(1/8)r_0^3-(1/8)r_0^2$$

$$q_1=(1/16)\left[(-3/2)(n/\omega)^2r_0^2(r_2-r_1-1)+(3/2)(n/\omega)^2r_0\times(r_2-r_1-1)-(45/32)(n/\omega)^2r_0^3-(3/8)r_0^3+(1/2)r_0^2-r_0\right]$$

$$q_2=(1/16)\left[(3/8)(n/\omega)^2r_0^2(r_2-r_1-1)+(9/16)(n/\omega)^2r_0^3-(3/32)r_0^4-(3/8)r_0^2+(3/8)r_0^3\right]$$

$$q_3=(1/16)\left[-(3/32)(n/\omega)^2r_0^3-(1/8)r_0^3\right]$$

$$q_4=0$$

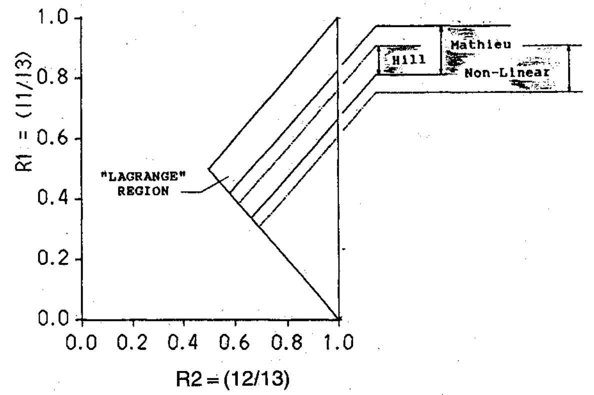

Fig. 2 Pitch instability zones for $r_0 = 0.33$ and $\omega = n$.

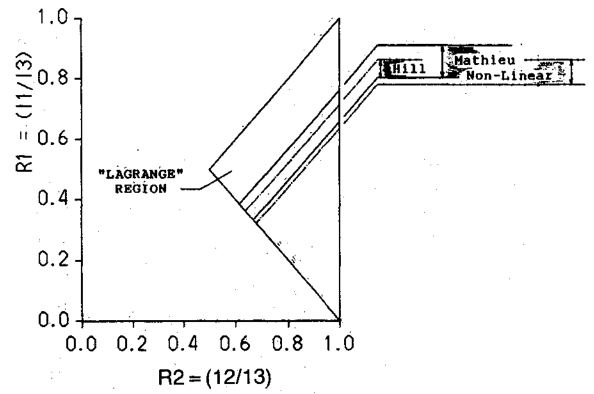

Fig. 3 Pitch instability zones for $r_0 = 0.23$ and $\omega = n$.

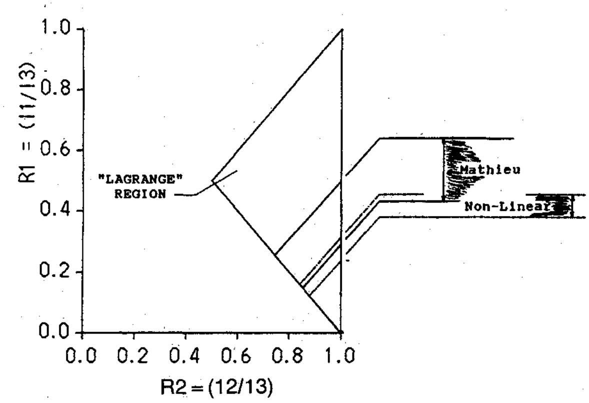

Fig. 4 Pitch instability zones for $r_0 = 0.33$ and $\omega = 1.4n$.

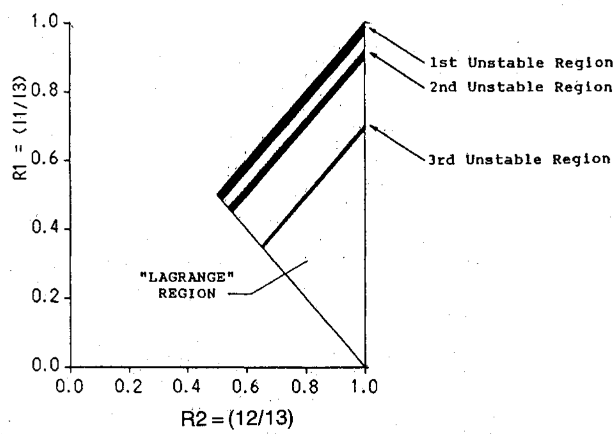

Fig. 5 Multiple Mathieu pitch instability zones for $r = 0.15$ and $\omega = 0.5n$.

larger than q_2 for typical space station characteristics. Both the Hill and Mathieu solutions predict the nonlinear instability, but neither defines it precisely in r_1, r_2 space.

The effect of decreasing the reduced mass parameter is seen by comparing Figs. 2 and 3. The lower line of the Hill and Mathieu solutions remains more or less fixed in r_1, r_2 space, but the upper boundary drops to reduce the size of the unstable region as the reduced mass becomes smaller. As M approaches zero, the region of moving mass instability shrinks to a line and then disappears.

The effect of increasing the frequency of the moving mass ω is to migrate the instability zone downward in the Lagrange region, as seen by comparing Fig. 4 to Fig. 2. A Hill region is not present on Fig. 4 because it vanishes for ω greater than approximately 1.3. Nonlinear instability was observed in the numerical integration up to approximately $\omega = 1.6$. The Mathieu solution continues to predict instability up to approximately $\omega = 1.9$.

It should also be noted that for frequencies $\omega < n$, additional resonance lines may appear corresponding to the second and higher Mathieu unstable regions. Figure 5 indicates the presence of three unstable subregions for $\omega = 0.5n$ and $r_0 = 0.15$.

Roll-Yaw Stability

A perturbation analysis known as the method of multiple scales is used to investigate roll and yaw stability.[15] Equations (7a) and (7b) can be written in the form

$$\ddot{u}_1 + \lambda_1 \dot{u}_2 + \alpha_1 u_1 = 0 \tag{11a}$$

$$\ddot{u}_2 - \lambda_2 \dot{u}_1 + \alpha_2 u_2 + 2\epsilon(f_{21}\dot{u}_1 + f_{22}\dot{u}_2)\sin(2\omega t) + 2\epsilon(f_{23}\dot{u}_1 + f_{24}u_2)\cos(2\omega t) = 0 \tag{11b}$$

where the state variables are redefined

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} y \\ r \end{bmatrix}$$

and the coefficients are

$$\lambda_1 = \left(\frac{I_{f3} - I_{f2} - I_{f1}}{I_{f1}}\right)n; \qquad \lambda_2 = \left(\frac{I_{f3} - I_{f2} - I_{f1}}{I_{f2}}\right)(1-\epsilon)n$$

$$\alpha_2 = 4\left[\left(\frac{I_{f3} - I_{f1}}{I_{f2}}\right) - \epsilon\left(\frac{I_{f3} - I_{f2} - I_{f1}}{I_{f2}}\right)\right]n^2$$

$$\alpha_1 = \left(\frac{I_{f3} - I_{f2}}{I_{f1}}\right)n^2$$

$$f_{21} = n\omega; \qquad f_{22} = \omega$$

$$f_{23} = -(1/2)\left(\frac{I_{f3} - I_{f2} - I_{f1}}{I_{f2}}\right)n; \qquad f_{24} = 2\left(\frac{I_{f3} - I_{f2} - I_{f1}}{I_{f2}}\right)n^2$$

$$\epsilon = \frac{r_0}{2r_2}$$

Development of Eqs. (11) is given in Ref. 13.

Following Nayfeh,[15] a power series solution in ϵ is assumed

$$u_m = u_{m0}(T_0, T_1) + \epsilon u_{m1}(T_0, T_1) \tag{12}$$

which includes two independent variables, or time scales, $T_0 = \epsilon^0 t$ and $T_1 = \epsilon t$. Only terms through first order in ϵ in the power series are required, since Eqs. (11) only contain ϵ to the first power. When Eq. (12) is substituted into Eqs. (11), a pair of order ϵ^0 and a pair of order ϵ^1 differential equations result.

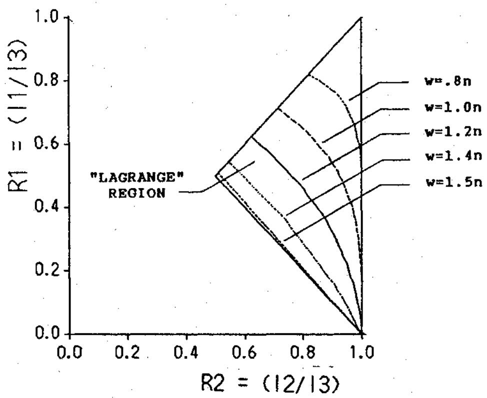

Fig. 6 Roll/yaw resonance lines for $r_0 = 0.33$ and $\omega = 0.5\,(\omega_1 + \omega_2)$.

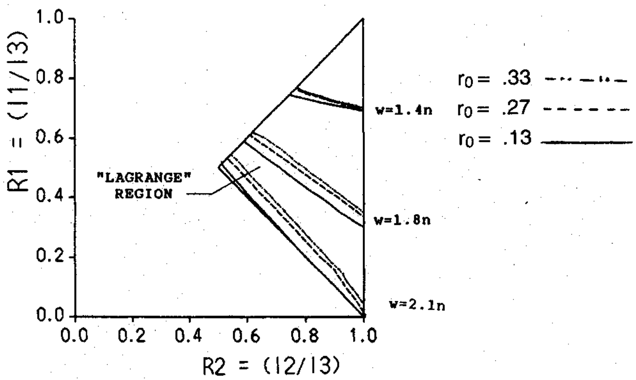

Fig. 7 Variation of roll/yaw resonance lines with parameter r_0 for $\omega = \omega_2$.

A solution to the order ϵ^0 pair can be found, since they are not dependent on the order ϵ^1 pair. Their solution, when substituted into the order ϵ^1 pair, produces secular terms whenever the frequency of the moving mass ω is such as to cause any of the driving terms on the right-hand side to modulate at frequency ω_1 or ω_2, the natural frequencies of the order ϵ^0 equations [i.e., the natural frequencies of Eq. (11) without the time varying terms]. Since secular terms grow indefinitely with time, this is a violation of the perturbation assumptions and an indication of station instability. The resonance phenomenon in this study occurs for the following conditions: 1) $\omega \cong 0.5(\omega_2 + \omega_1)$, 2) $\omega \cong 0.5(\omega_2 - \omega_1)$, 3) $\omega \cong \omega_2$, and 4) $\omega \cong \omega_1$. Figure 6 shows the locus of configurations within the Lagrange region that produce resonance condition 1 where ω is the parameter and r_0, the normalized reduced mass, is equal to 0.33, the upper limit considered. The loci for resonance conditions 2 and 4, which are not presented, appear in the Lagrange region for $\omega < n$. The loci for condition 3, shown in Fig. 7, appear for $\omega > n$. Thus, at frequencies above and below orbital frequency, multiple resonances from conditions 1–4 can appear. The highest frequency at which resonance appears within the Lagrange region is at $\omega = \omega_2 \cong 2.3$.

When r_0 is reduced, there is a slight shift in the resonance lines, as indicated in Fig. 7 for condition 3. The shift is less for condition 1.

It should be realized that the unstable motion associated with these resonance lines is not confined to the lines themselves. Mathematical approximation of the bandwidth around a resonance line is described by Nayfeh (see Ref. 15, pp. 336, 337). The bandwidths are a function of ϵ and, therefore, depend on r_0 and r_2. Although r_0 is a chosen constant, the value of r_2 varies along the resonance lines. Therefore, the bandwidth will have a tapered appearance across the Lagrange region. Numerical results were produced to verify the existence of these bands.

Conclusions

The equations of attitude motion have been investigated for a gravity-gradient stabilized space station that includes periodic motion of an elevator-type mechanism, equipment, or crew along one of the principal axes of the station. The linearized equations contain periodic coefficients that cause parametric excitation and instability when the elevator period is in the vicinity of the orbital period. The impact is to place restrictions on station shape; specifically, eliminating portions of the Lagrange region that define stable shapes for a rigid body model. The exact shape and location of the elevator-induced unstable regions depend on the mass of the elevator as well as its amplitude and frequency of oscillation. Numerical integration of the nonlinear equations of motion for selected initial conditions verifies the existence of these instabilities for large angular motions.

References

[1]Spacecraft Gravitational Torques," NASA SP-8024, 1969.

[2]Debra, D. B., and Delp, R. H., "Satellite Stability and Natural Frequencies in a Circular Orbit," *Journal of Astronautical Sciences,* Vol. 8, No. 1, 1961.

[3]Hughes, P. C., *Spacecraft Attitude Dynamics,* Wiley, NY, 1986.

[4]Kane, T. R., "Attitude Stability of Earth-Pointing Satellites," *AIAA Journal,* Vol. 3, No. 4, 1965, pp. 726–731.

[5]Breakwell, J. V., and Pringle, R., Jr., "Nonlinear Resonance Affecting Gravity-Gradient Stability," *Astrodynamics,* edited by M. Lunc, Gauthier-Villars, Paris, 1966.

[6]Modi, V. J., and Brereton, R. C., "Libration Analysis of a Dumbbell Satellite Using the WKBJ Method," *Journal of Applied Mechanics,* Vol. 33, No. 3, 1966, pp. 676–678.

[7]Beletskii, V. V., "Motion of an Artificial Satellite About its Center of Mass," NASA TT-429, 1966.

[8]Chubb, W. B., Kennel, H. F., Rupp, C. C. and Seltzer, S. M., "Flight Performance of Skylab Attitude and Pointing Control System," *Journal of Spacecraft,* Vol. 12, 1975, pp. 220–227.

[9]Thomson, W. T., and Fung, Y. C., "Instability of Spinning Space Station Due to Crew Motion," *AIAA Journal,* Vol. 3, No. 6, 1965, pp. 1082–1087.

[10]Salimov, G. R., "Stability of Rotating Space Station Containing a Moving Element," *Mechanics of Solids,* Vol. 10, No. 5, 1975, pp. 41–45.

[11]Wie, B., Hu A., and Singh, R., "Multi-Body Interaction Effects on Space Station Attitude Control and Momentum Management," *Proceedings of AIAA Guidance, Navigation and Control Conference,* Part 1, AIAA, Washington, DC, Aug. 1989, pp. 770–780.

[12]Li, D., and Likins, P. W., "Dynamics of a Multibody System with Relative Translation on Curved, Flexible Tracks," *Journal of Guidance, Control, and Dynamics,* Vol. 10, No. 3, 1987, pp. 299–306.

[13]Williams, T. E., "An Analysis of the Attitude Stability of a Space Station Subject to Parametric Excitation of Periodic Mass Motion," M.S. Thesis, Air Force Inst. of Tech., AFIT/GSO/AA/88D-2, Dec. 1988.

[14]Hayashi, C., *Nonlinear Oscillation in Physical Systems,* McGraw-Hill, New York, 1964.

[15]Nayfeh, A. H., and Mook, D. T., *Nonlinear Oscillations,* Wiley, New York, 1979.